Frequency Selective Sinusoidal Order Estimation

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Frequency selective sinusoidal order estimation

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Proposed is a frequency selective (FS) subspace-based method for determining the model order. A study is made of its performance when applied to estimating the number of sinusoids in white noise. Employing an FS-version of the ESPRIT algorithm, the recent ESTER model order estimation algorithm is extended to allow for the model order estimation on a frequency subset.

Introduction: Estimating the order of a model is a central, yet commonly overlooked, problem in parameter estimation, with the majority of the literature assuming prior knowledge of the model order. In many cases, however, the order cannot be known *a priori* and may change over time. The prevalent methods for estimating the model order are the minimum description length (MDL) [1, 2], the Akaike information criterion (AIC) [3], and the maximum *a posteriori* (MAP) rule [4]. These methods are based on statistical models of the observed signal, such as the observation noise being white and Gaussian distributed. From these models, a regularised estimation criterion is devised that is composed of a log-likelihood term and an order-dependent penalty term. We refer the reader to [5] for an overview of such statistical methods. The problem that we are here concerned with is that of sinusoidal order estimation, i.e. determining the number of sinusoids in noise. This problem is treated in much detail from a statistical point of view in [4] and is also exemplified in [5]. Mathematically, the problem can be stated as follows. Consider a complex signal consisting of (a possibly large number of) complex sinusoids having frequencies $\{\omega_l\}$ which is corrupted by an additive noise, $w(n)$, for $n = 0, \ldots, N-1$, i.e.

$$x(n) = \sum_{l=1}^{P} A_l e^{j(\omega_l n + \phi_l) + \beta_l n} + \omega(n) \tag{1}$$

where $A_l > 0$, ϕ_l and β_l are the amplitude, the phase and damping of the lth sinusoid. Here, $\omega(n)$, is assumed to be white complex symmetric zero-mean noise. It is noted that the sinusoids may be damped. Herein, we are interested in the problem of estimating the model order, $L \ll P$, in a specific frequency band of interest specified by a subset of discrete Fourier transform bases. In this Letter, we propose an estimation criterion based on a frequency selective (FS) subspace technique.

Algorithm: Following the notation in [6–8], we note that it is possible to form a frequency selective data model allowing for the approximation

$$\mathbf{Y}\Pi_{\mathbf{U}}^{\perp} \simeq \mathbf{A}_{\ell}\mathbf{X}_{\ell}\Pi_{\mathbf{U}}^{\perp} \tag{2}$$

where $\mathbf{Y} \in \mathbb{C}^{S \times M}$ is the FS data matrix, $\Pi_{\mathbf{U}}^{\perp}$ a projection onto the space orthogonal to the $S \times M$ Vandermonde matrix $\mathbf{U}$, formed from a subset of discrete Fourier transform bases, $\mathbf{A}_{\ell}$ a $S \times \ell$ Vandermonde matrix formed from the assumed ℓ modes in the selected frequency range, and $\mathbf{X}_{\ell} \in \mathbb{C}^{\ell \times M}$ a matrix formed from the Fourier transformed modes. (In the interest of brevity, we here simply state the approximative expression (2), referring the reader to [6–8] for further details on the definitions and derivations.) As in [7], $M \geq L + S$ denotes the number of selected (possibly consecutive) frequency grid points and the user parameter $S \in (\lfloor M/3 \rfloor, \lfloor M/2 \rfloor)$, where $\lfloor x \rfloor$ denotes the integer part of x. We note that using (2), one may form a (possibly weighted [8]) estimate of the unknown modes. Herein, reminiscent to the ESTER algorithm proposed in [9], we propose to form a cost function, $J(\ell)$, based on the goodness of the fit in (2) for a generic order ℓ, i.e.

$$J(\ell) = \|\mathbf{Y}\Pi_{\mathbf{U}}^{\perp} - \mathbf{A}_{\ell}\mathbf{X}_{\ell}\Pi_{\mathbf{U}}^{\perp}\|_2^2 \tag{3}$$

where $\|\cdot\|_2$ denotes the 2-norm. We then propose to estimate the model order as

$$\hat{L} = \arg\min_{\ell} J(\ell) \tag{4}$$

It is worth noting that, unlike commonly used statistical methods, the method does not depend on the noise probability density function. Furthermore, we note that should the full frequency range be used in forming (3), (4) will coincide with the ESTER estimate.

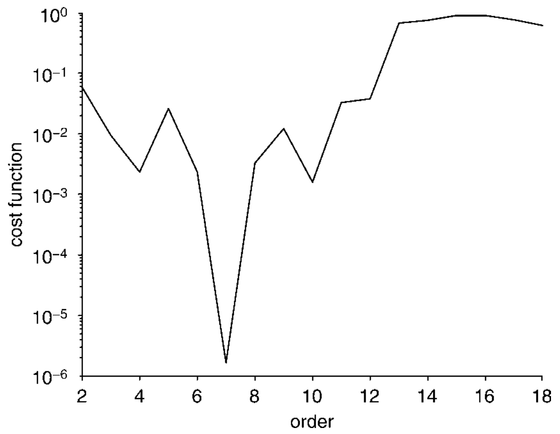

Fig. 1 *Example of cost function for various model orders with L = 7*

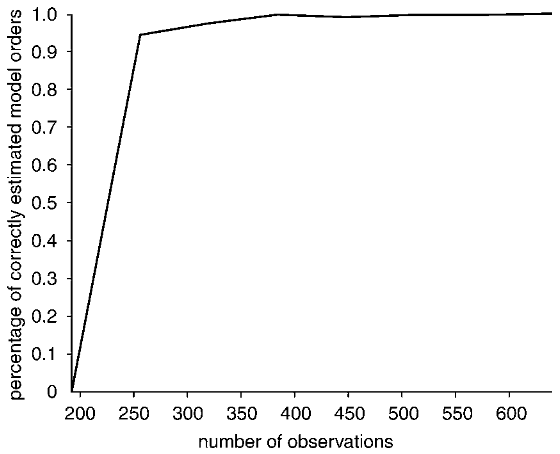

Fig. 2 *Percentage of correctly estimated model orders against number of observations*

Results: Fig. 1 is an illustrative example of the cost function of the proposed method. Here, the data consists of $P = 107$ real-valued sinusoids corrupted by real-valued white noise, whereof $L = 7$ sinusoids reside in the frequency region $\omega \in [0.19, 0.28]$. The signal length is 512 samples, $M = 46$ and $S = 20$. In Fig. 2, we examine the percentage of correctly estimated model orders against the number of observations. The results are obtained using 2000 Monte-Carlo simulations of the above data set, with each simulation randomising the initial phases and the corrupting noise sequence. Here, $S = \lfloor M/2 \rfloor$ and $SNR = -8$ dB, where SNR is defined as the power of the L sinusoids to the noise and interference (in this case, the P–L sinusoids).

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